

## 8<sup>th</sup> Grade Algebra I

The fundamental purpose of 8<sup>th</sup> Grade Algebra I is to formalize and extend the mathematics that students learned through the end of seventh grade. The critical areas, called units, deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. In addition, the units will introduce methods for analyzing and using quadratic functions, including manipulating expressions for them, and solving quadratic equations. Students understand and apply the Pythagorean theorem, and use quadratic functions to model and solve problems. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

This course differs from High School Algebra I in that it contains content from 8<sup>th</sup> grade. While coherence is retained, in that it logically builds from the Accelerated 7<sup>th</sup> Grade, the additional content when compared to the high school course demands a faster pace for instruction and learning.

**Critical Area 1:** Work with quantities and rates, including simple linear expressions and equations forms the foundation for this unit. Students use units to represent problems algebraically and graphically, and to guide the solution of problems. Student experience with quantity provides a foundation for the study of expressions, equations, and functions. This unit builds on earlier experiences with equations by asking students to analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

**Critical Area 2:** Building on earlier work with linear relationships, students learn function notation and language for describing characteristics of functions, including the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students explore systems of equations and inequalities, and they find and interpret their solutions. Students build on and informally extend their understanding of integral exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

**Critical Area 3:** Students use regression techniques to describe relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

**Critical Area 4:** In this unit, students build on their knowledge from unit 2, where they extended the laws of exponents to rational exponents. Students apply this new understanding of number and strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

**Critical Area 5:** In preparation for work with quadratic relationships students explore distinctions between rational and irrational numbers. They consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows  $x+1 = 0$  to have a solution. Formal work with complex numbers comes in Algebra II. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

Units	Includes Standard Clusters*	Mathematical Practice Standards
<p><b>Unit 1</b> Relationships Between Quantities and Reasoning with Equations</p>	<ul style="list-style-type: none"> <li>Reason quantitatively and use units to solve problems.</li> <li>Interpret the structure of expressions.</li> <li>Create equations that describe numbers or relationships.</li> <li>Understand solving equations as a process of reasoning and explain the reasoning.</li> <li>Solve equations and inequalities in one variable.</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p>
<p><b>Unit 2</b> Linear and Exponential Relationships</p>	<ul style="list-style-type: none"> <li>Extend the properties of exponents to rational exponents.</li> <li>Analyze and solve linear equations and pairs of simultaneous linear equations.</li> <li>Solve systems of equations.</li> <li>Represent and solve equations and inequalities graphically</li> <li>Define, evaluate, and compare functions.</li> <li>Understand the concept of a function and use function notation.</li> <li>Use functions to model relationships between quantities.</li> <li>Interpret functions that arise in applications in terms of a context.</li> <li>Analyze functions using different representations.</li> <li>Build a function that models a relationship between two quantities.</li> <li>Build new functions from existing functions.</li> <li>Construct and compare linear, quadratic, and exponential models and solve problems.</li> <li>Interpret expressions for functions in terms of the situation they model.</li> </ul>	<p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>
<p><b>Unit 3</b> Descriptive Statistics</p>	<ul style="list-style-type: none"> <li>Summarize, represent, and interpret data on a single count or measurement variable.</li> <li>Investigate patterns of association in bivariate data.</li> <li>Summarize, represent, and interpret data on two categorical and quantitative variables.</li> <li>Interpret linear models.</li> </ul>	

\*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

Units	Includes Standard Clusters	Mathematical Practice Standards
<p style="text-align: center;"><b>Unit 4</b> Expressions and Equations</p>	<ul style="list-style-type: none"> <li>• Interpret the structure of expressions.</li> <li>• Write expressions in equivalent forms to solve problems.</li> <li>• Perform arithmetic operations on polynomials.</li> <li>• Create equations that describe numbers or relationships.</li> <li>• Solve equations and inequalities in one variable.</li> <li>• Solve systems of equations.</li> </ul>	
<p style="text-align: center;"><b>Unit 5</b> Quadratics Functions and Modeling</p>	<ul style="list-style-type: none"> <li>• Use properties of rational and irrational numbers.</li> <li>• Understand and apply the Pythagorean theorem.</li> <li>• Interpret functions that arise in applications in terms of a context.</li> <li>• Analyze functions using different representations.</li> <li>• Build a function that models a relationship between two quantities.</li> <li>• Build new functions from existing functions.</li> <li>• Construct and compare linear, quadratic and exponential models and solve problems.</li> </ul>	

## Unit 1: Relationships between Quantities and Reasoning with Equations

Work with quantities and rates, including simple linear expressions and equations forms the foundation for this unit. Students use units to represent problems algebraically and graphically, and to guide the solution of problems. Student experience with quantity provides a foundation for the study of expressions, equations, and functions. This unit builds on earlier experiences with equations by asking students to analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

Unit 1: Relationships between Quantities and Reasoning with Equations	
Clusters with Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Reason quantitatively and use units to solve problems.</li> </ul> <p><i>Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.<sup>12</sup></i></p>	<p>N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.</p> <p>N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.</p> <p>N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.</p>
<ul style="list-style-type: none"> <li>Interpret the structure of expressions.</li> </ul> <p><i>Limit to linear expressions and to exponential expressions with integer exponents.</i></p>	<p>A.SSE.1 Interpret expressions that represent a quantity in terms of its context.*</p> <ol style="list-style-type: none"> <li>Interpret parts of an expression, such as terms, factors, and coefficients.</li> <li>Interpret complicated expressions by viewing one or more of their parts as a single entity. <i>For example, interpret <math>P(1+r)^n</math> as the product of <math>P</math> and a factor not depending on <math>P</math>.</i></li> </ol>
<ul style="list-style-type: none"> <li>Create equations that describe numbers or relationships.</li> </ul> <p><i>Limit A.CED.1 and A.CED.2 to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs. Limit A.CED.3 to linear equations and inequalities. Limit A.CED.4 to formulas which are linear in the variables of interest.</i></p>	<p>A.CED.1 Create equations and inequalities in one variable and use them to solve problems. <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i></p> <p>A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</p> <p>A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. <i>For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</i></p> <p>A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. <i>For example, rearrange Ohm's law <math>V = IR</math> to highlight resistance <math>R</math>.</i></p>
<ul style="list-style-type: none"> <li>Understand solving equations as a process of reasoning and explain the reasoning.</li> </ul> <p><i>Students should focus on and master A.REI.1 for linear equations and be able to extend and apply their reasoning to other types of equations in future units and courses. Students will solve exponential equations in Algebra II.</i></p>	<p>A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.</p>

Unit 1: Relationships between Quantities and Reasoning with Equations	
Clusters with Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Solve equations and inequalities in one variable.</li> </ul> <p><i>Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as <math>5^x = 125</math> or <math>2^x = \frac{1}{16}</math>.</i></p>	<p>A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.</p>

## Unit 2: Linear and Exponential Functions

Building on earlier work with linear relationships, students learn function notation and language for describing characteristics of functions, including the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students explore systems of equations and inequalities, and they find and interpret their solutions. Students build on and informally extend their understanding of integral exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Unit 2: Linear and Exponential Functions	
Clusters with Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Extend the properties of exponents to rational exponents.</li> </ul> <p><i>In implementing the standards in curriculum, these standards should occur before discussing exponential models with continuous domains.</i></p>	<p>N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. <i>For example, we define <math>5^{1/3}</math> to be the cube root of 5 because we want <math>(5^{1/3})^3 = 5^{(1/3) \cdot 3}</math> to hold, so <math>(5^{1/3})^3</math> must equal 5.</i></p> <p>N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.</p>
<ul style="list-style-type: none"> <li>Analyze and solve linear equations and pairs of simultaneous linear equations.</li> </ul> <p><i>While this content is likely subsumed by A.REI.3, 5, and 6, it could be used for scaffolding instruction to the more sophisticated content found there.</i></p>	<p>8.EE.8 Analyze and solve pairs of simultaneous linear equations.</p> <ol style="list-style-type: none"> <li>Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.</li> <li>Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. <i>For example, <math>3x + 2y = 5</math> and <math>3x + 2y = 6</math> have no solution because <math>3x + 2y</math> cannot simultaneously be 5 and 6.</i></li> <li>Solve real-world and mathematical problems leading to two linear equations in two variables. <i>For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.</i></li> </ol>
<ul style="list-style-type: none"> <li>Solve systems of equations.</li> </ul> <p><i>Include cases where two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution).</i></p>	<p>A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.</p> <p>A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.</p>

## Unit 2: Linear and Exponential Functions

Clusters with Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Represent and solve equations and inequalities graphically.</li> </ul> <p><i>For A.REI.10 focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses. For A.REI.11, focus on cases where <math>f(x)</math> and <math>g(x)</math> are linear or exponential.</i></p>	<p>A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).</p> <p>A.REI.11 Explain why the <math>x</math>-coordinates of the points where the graphs of the equations <math>y = f(x)</math> and <math>y = g(x)</math> intersect are the solutions of the equation <math>f(x) = g(x)</math>; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where <math>f(x)</math> and/or <math>g(x)</math> are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*</p> <p>A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.</p>
<ul style="list-style-type: none"> <li>Define, evaluate, and compare functions.</li> </ul> <p><i>While this content is likely subsumed by F.IF.1-3 and F.IF.7a, it could be used for scaffolding instruction to the more sophisticated content found there.</i></p>	<p>8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.</p> <p>8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.</i></p> <p>8.F.3 Interpret the equation <math>y = mx + b</math> as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. <i>For example, the function <math>A = s^2</math> giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.</i></p>
<ul style="list-style-type: none"> <li>Understand the concept of a function and use function notation.</li> </ul> <p><i>Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses.</i></p> <p><i>Constrain examples to linear functions and exponential functions having integral domains. In F.IF.3, draw connection to F.BF.2, which requires students to write arithmetic and geometric sequences.</i></p>	<p>F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If <math>f</math> is a function and <math>x</math> is an element of its domain, then <math>f(x)</math> denotes the output of <math>f</math> corresponding to the input <math>x</math>. The graph of <math>f</math> is the graph of the equation <math>y = f(x)</math>.</p> <p>F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.</p> <p>F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. <i>For example, the Fibonacci sequence is defined recursively by <math>f(0) = f(1) = 1</math>, <math>f(n+1) = f(n) + f(n-1)</math> for <math>n \geq 1</math>.</i></p>
<ul style="list-style-type: none"> <li>Use functions to model relationships between quantities.</li> </ul> <p><i>While this content is likely subsumed by F.IF.4 and F.BF.1a, it could be used for scaffolding instruction to the more sophisticated content found there.</i></p>	<p>8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two <math>(x, y)</math> values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</p> <p>8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.</p>

## Unit 2: Linear and Exponential Functions

Clusters with Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Interpret functions that arise in applications in terms of a context.</li> </ul> <p><i>For F.IF.4 and 5, focus on linear and exponential functions. For F.IF.6, focus on linear functions and exponential functions whose domain is a subset of the integers. Unit 5 in this course and Algebra II course address other types of functions.</i></p>	<p>F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i>*</p> <p>F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For example, if the function <math>h(n)</math> gives the number of person-hours it takes to assemble <math>n</math> engines in a factory, then the positive integers would be an appropriate domain for the function.</i>*</p> <p>F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*</p>
<ul style="list-style-type: none"> <li>Analyze functions using different representations.</li> </ul> <p><i>For F.IF.7a, 7e, and 9 focus on linear and exponentials functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as <math>y=3^n</math> and <math>y=100 \cdot 2^n</math>.</i></p>	<p>F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*</p> <ol style="list-style-type: none"> <li>Graph linear and quadratic functions and show intercepts, maxima, and minima.</li> <li>Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</li> </ol> <p>F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i></p>
<ul style="list-style-type: none"> <li>Build a function that models a relationship between two quantities.</li> </ul> <p><i>Limit F.BF.1a, 1b, and 2 to linear and exponential functions. In F.BF.2, connect arithmetic sequences to linear functions and geometric sequences to exponential functions in F.BF.2.</i></p>	<p>F.BF.1 Write a function that describes a relationship between two quantities.*</p> <ol style="list-style-type: none"> <li>Determine an explicit expression, a recursive process, or steps for calculation from a context.</li> <li>Combine standard function types using arithmetic operations. <i>For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</i></li> </ol> <p>F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*</p>
<ul style="list-style-type: none"> <li>Build new functions from existing functions.</li> </ul> <p><i>Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its <math>y</math>-intercept.</i></p> <p><i>While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.</i></p>	<p>F.BF.3 Identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>k f(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math> (both positive and negative); find the value of <math>k</math> given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. <i>Include recognizing even and odd functions from their graphs and algebraic expressions for them.</i></p>

Unit 2: Linear and Exponential Functions	
Clusters with Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Construct and compare linear, quadratic, and exponential models and solve problems.</li> </ul> <p><i>For F.LE.3, limit to comparisons between linear and exponential models.</i></p>	<p>F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.</p> <ol style="list-style-type: none"> <li>Prove that linear functions grow by equal differences over equal intervals; and that exponential functions grow by equal factors over equal intervals.</li> <li>Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.</li> <li>Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.</li> </ol> <p>F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).</p> <p>F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.</p>
<ul style="list-style-type: none"> <li>Interpret expressions for functions in terms of the situation they model.</li> </ul> <p><i>Limit exponential functions to those of the form <math>f(x) = b^x + k</math>.</i></p>	<p>F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.</p>

## Unit 3: Descriptive Statistics

Students use regression techniques to describe relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Unit 3: Descriptive Statistics	
Clusters with Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Summarize, represent, and interpret data on a single count or measurement variable.</li> </ul> <p><i>In grades 6 – 7, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.</i></p>	<p>S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).</p> <p>S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.</p> <p>S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).</p>
<ul style="list-style-type: none"> <li>Investigate patterns of association in bivariate data.</li> </ul> <p><i>While this content is likely subsumed by S.ID.6-9, it could be used for scaffolding instruction to the more sophisticated content found there.</i></p>	<p>8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.</p> <p>8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.</p> <p>8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. <i>For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.</i></p> <p>8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. <i>For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?</i></p>
<ul style="list-style-type: none"> <li>Summarize, represent, and interpret data on two categorical and quantitative variables.</li> </ul> <p><i>Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.</i></p> <p><i>S.ID.6b should be focused on linear models, but may be used to preface quadratic functions in the Unit 6 of this course.</i></p>	<p>S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.</p> <p>S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.</p> <ol style="list-style-type: none"> <li>Fit a function to the data; use functions fitted to data to solve problems in the context of the data. <i>Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.</i></li> <li>Informally assess the fit of a function by plotting and analyzing residuals.</li> <li>Fit a linear function for a scatter plot that suggests a linear association.</li> </ol>

Unit 3: Descriptive Statistics	
Clusters with Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Interpret linear models.</li> </ul> <p><i>Build on students' work with linear relationship and; introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure how well the data fit the relationship. The important distinction between a statistical relationship and a cause-and-effect relationship arises in S.ID.9.</i></p>	<p>S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.</p> <p>S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.</p> <p>S.ID.9 Distinguish between correlation and causation.</p>

## Unit 4: Expressions and Equations

In this unit, students build on their knowledge from unit 2, where they extended the laws of exponents to rational exponents. Students apply this new understanding of number and strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

Unit 4: Expressions and Equations	
Clusters with Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Interpret the structure of expressions.</li> </ul> <p><i>Focus on quadratic and exponential expressions. For A.SSE.1b, exponents are extended from integer found in Unit 1 to rational exponents focusing on those that represent square roots and cube roots.</i></p>	<p>A.SSE.1 Interpret expressions that represent a quantity in terms of its context.*</p> <ol style="list-style-type: none"> <li>Interpret parts of an expression, such as terms, factors, and coefficients.</li> <li>Interpret complicated expressions by viewing one or more of their parts as a single entity. <i>For example, interpret <math>P(1+r)^n</math> as the product of <math>P</math> and a factor not depending on <math>P</math>.</i></li> </ol> <p>A.SSE.2 Use the structure of an expression to identify ways to rewrite it. <i>For example, see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, thus recognizing it as a difference of squares that can be factored as <math>(x^2 - y^2)(x^2 + y^2)</math>.</i></p>
<ul style="list-style-type: none"> <li>Write expressions in equivalent forms to solve problems.</li> </ul> <p><i>Consider extending this unit to include the relationship between properties of logarithms and properties of exponents.</i></p>	<p>A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*</p> <ol style="list-style-type: none"> <li>Factor a quadratic expression to reveal the zeros of the function it defines.</li> <li>Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.</li> <li>Use the properties of exponents to transform expressions for exponential functions. <i>For example the expression <math>1.15^t</math> can be rewritten as <math>(1.15^{1/12})^{12t} \approx 1.012^{12t}</math> to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.</i></li> </ol>
<ul style="list-style-type: none"> <li>Perform arithmetic operations on polynomials.</li> </ul> <p><i>Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of <math>x</math>.</i></p>	<p>A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.</p>
<ul style="list-style-type: none"> <li>Create equations that describe numbers or relationships.</li> </ul> <p><i>Extend work on linear and exponential equations in Unit 1 to include quadratic equations. Extend A.CED.4 to formulas involving squared variables.</i></p>	<p>A.CED.1 Create equations and inequalities in one variable and use them to solve problems. <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i></p> <p>A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</p> <p>A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. <i>For example, rearrange Ohm's law <math>V = IR</math> to highlight resistance <math>R</math>.</i></p>

Unit 4: Expressions and Equations	
Clusters with Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Solve equations and inequalities in one variable.</li> </ul> <p><i>Students should learn of the existence of the complex number system, but will not solve quadratics with complex solutions until Algebra II.</i></p>	<p>A.REI.4 Solve quadratic equations in one variable.</p> <ol style="list-style-type: none"> <li>Use the method of completing the square to transform any quadratic equation in <math>x</math> into an equation of the form <math>(x - p)^2 = q</math> that has the same solutions. Derive the quadratic formula from this form.</li> <li>Solve quadratic equations by inspection (e.g., for <math>x^2 = 49</math>), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as <math>a \pm bi</math> for real numbers <math>a</math> and <math>b</math>.</li> </ol>
<ul style="list-style-type: none"> <li>Solve systems of equations.</li> </ul> <p><i>Include systems consisting of one linear and one quadratic equation. Include systems that lead to work with fractions. For example, finding the intersections between <math>x^2 + y^2 = 1</math> and <math>y = (x+1)/2</math> leads to the point <math>(3/5, 4/5)</math> on the unit circle, corresponding to the Pythagorean triple <math>3^2 + 4^2 = 5^2</math>.</i></p>	<p>A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. <i>For example, find the points of intersection between the line <math>y = -3x</math> and the circle <math>x^2 + y^2 = 3</math>.</i></p>

## Unit 5: Quadratic Functions and Modeling

In preparation for work with quadratic relationships students explore distinctions between rational and irrational numbers. They consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows  $x+1 = 0$  to have a solution. Formal work with complex numbers comes in Algebra II. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

Unit 5: Quadratic Functions and Modeling	
Clusters with Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Use properties of rational and irrational numbers.</li> </ul> <p><i>Connect N.RN.3 to physical situations, e.g., finding the perimeter of a square of area 2.</i></p>	<p>N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.</p>
<ul style="list-style-type: none"> <li>Understand and apply the Pythagorean theorem.</li> </ul> <p><i>Discuss applications of the Pythagorean theorem and its connections to radicals, rational exponents, and irrational numbers.</i></p>	<p>8.G.6 Explain a proof of the Pythagorean theorem and its converse.</p> <p>8.G.7 Apply the Pythagorean theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.</p> <p>8.G.8 Apply the Pythagorean theorem to find the distance between two points in a coordinate system.</p>
<ul style="list-style-type: none"> <li>Interpret functions that arise in applications in terms of a context.</li> </ul> <p><i>Focus on quadratic functions; compare with linear and exponential functions studied in Unit 2.</i></p>	<p>F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*</i></p> <p>F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For example, if the function <math>h(n)</math> gives the number of person-hours it takes to assemble <math>n</math> engines in a factory, then the positive integers would be an appropriate domain for the function.*</i></p> <p>F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*</p>

Unit 5: Quadratic Functions and Modeling	
Clusters with Instructional Notes	Common Core State Standards
<ul style="list-style-type: none"> <li>Analyze functions using different representations.</li> </ul> <p><i>For F.IF.7b, compare and contrast absolute value, step and piecewise-defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range, and usefulness when examining piecewise-defined functions. Note that this unit, and in particular in F.IF.8b, extends the work begun in Unit 2 on exponential functions with integral exponents. For F.IF.9, focus on expanding the types of functions considered to include, linear, exponential, and quadratic.</i></p> <p><i>Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored.</i></p>	<p>F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★</p> <ol style="list-style-type: none"> <li>Graph linear and quadratic functions and show intercepts, maxima, and minima.</li> <li>Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</li> </ol> <p>F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</p> <ol style="list-style-type: none"> <li>Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.</li> <li>Use the properties of exponents to interpret expressions for exponential functions. <i>For example, identify percent rate of change in functions such as <math>y = (1.02)^t</math>, <math>y = (0.97)^t</math>, <math>y = (1.01)^{2t}</math>, <math>y = (1.2)^{t/10}</math>, and classify them as representing exponential growth or decay.</i></li> </ol> <p>F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i></p>
<ul style="list-style-type: none"> <li>Build a function that models a relationship between two quantities.</li> </ul> <p><i>Focus on situations that exhibit a quadratic relationship.</i></p>	<p>F.BF.1 Write a function that describes a relationship between two quantities.★</p> <ol style="list-style-type: none"> <li>Determine an explicit expression, a recursive process, or steps for calculation from a context.</li> <li>Combine standard function types using arithmetic operations. <i>For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</i></li> </ol>
<ul style="list-style-type: none"> <li>Build new functions from existing functions.</li> </ul> <p><i>For F.BF.3, focus on quadratic functions, and consider including absolute value functions. For F.BF.4a, focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as <math>f(x) = x^2</math>, <math>x &gt; 0</math></i></p>	<p>F.BF.3 Identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>k f(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math> (both positive and negative); find the value of <math>k</math> given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. <i>Include recognizing even and odd functions from their graphs and algebraic expressions for them.</i></p> <p>F.BF.4 Find inverse functions.</p> <ol style="list-style-type: none"> <li>Solve an equation of the form <math>f(x) = c</math> for a simple function <math>f</math> that has an inverse and write an expression for the inverse. <i>For example, <math>f(x) = 2x^3</math> for <math>x &gt; 0</math> or <math>f(x) = (x+1)/(x-1)</math> for <math>x \neq 1</math>.</i></li> </ol>
<ul style="list-style-type: none"> <li>Construct and compare linear, quadratic, and exponential models and solve problems.</li> </ul> <p><i>Compare linear and exponential growth to growth of quadratic growth.</i></p>	<p>F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.</p>

## KML Federation Notes:

### Page 1:

Note the emphasis on linear, quadratic, and exponential regression. Also, note the emphasis on students working with absolute value, step, and piece-wise functions.

### Page 2:

Note the emphasis on statistics. "Summarize, represent, and interpret data on two categorical variables" involves using contingency (Pivot) tables and side-by-side and segmented bar graphs.

### Page 4:

"A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales."

It should be noted that "labels" also includes appropriate titles to graphs involving real-world relationships.

### Page 5:

"A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters."

"Equations with coefficients represented by letters" are also known as literal equations or formulas.

### In general:

-Consideration should be made to incorporating some preliminary work with logarithms (as inverses of exponential functions, solving log equations, introduction to log properties). These standards mention including these topics.